Insight Academy of Canada

Course Outline

Course Title	Calculus and Vectors, Grade 12, University Preparation (MCV4U)
Grade	12
Course Type	University
Course Code	MCV4U
Credit Value	1.0
Curriculum Policy Documents	The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007 (revised) Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, 2010
Prerequisite(s)	MCR3U, Functions, University Preparation The new Advanced Functions course (MHF4U) must be taken prior to or concurrently with Calculus and Vectors (MCV4U)

Course Description

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

Overall Curriculum Expectations

A. Rate of Change By the end of this course, students will:	
A1	Demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit;
A2	Graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative;
A3	Verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.
B. Derivatives and Their Applications By the end of this course, students will:	
B1	Make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;
B2	Solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.
C. Geometry and Algebra of Vectors By the end of this course, students will:	
C1	Demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications;
C2	Perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications;
C3	Distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space;

C4	Represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.

Outline of Course Content

Unit No.	Unit Title	Instructional Time	Overall Expectations
1	Calculus: An Introduction	11 hours	A1
2	Derivatives	11 hours	A3
3	Applications of Derivatives	11 hours	B1, B2
4	Curve Sketching	11 hours	B1
5	Derivatives of Exponential and Trigonometric Functions	11 hours	A2, B2
6	Vectors: An Introduction	14 hours	C1, C2
7	Applications of Vectors	15 hours	C1, C2
8	Equations of Lines and Planes	13 hours	C4
9	Relationships Between Points, Lines, and Planes	13 hours	C3, C4
Total Instructional Time		**110 HOURS**	
10	Final Exam	2.5 hours	All

Unit Descriptions

The entire course is delivered online

Unit 1: Calculus: An Introduction

A variety of mathematical operations with functions are needed in order to do the calculus of this course. This unit begins with students developing a better understanding of these essential concepts. Students will then deal with rates of change problems and the limit concept. While the concept of a limit involves getting close to a value but never getting to the value, often the limit of a function can be determined by substituting the value of interest for the variable in the function. Students will work with several examples of this concept. The indeterminate form of a limit involving factoring, rationalization, change of variables and one-sided limits are all included in the exercises undertaken next in this unit. To further investigate the concept of a limit, the unit briefly looks at the relationship between a secant line and a tangent line to a curve. To this point in the course students have been given a fixed point and have been asked to find the tangent slope at that value, in this section of the unit students will determine a tangent slope function similar to what they had done with a secant slope function. Sketching the graph of a derivative function is the final skill and topic.

Specific Expectations: A1.1, A1.2, A1.3, A1.4, A1.5, A1.6

Unit 2: Derivatives

The concept of a derivative is, in essence, a way of creating a short cut to determine the tangent line slope function that would normally require the concept of a limit. Once patterns are seen from the evaluation of limits, rules can be established to simplify what must be done to determine this slope function. This unit begins by examining those rules including: the power rule, the product rule, the quotient rule and the chain rule followed by a study of the derivatives of composite functions. The next section is dedicated to finding the derivative of relations that cannot be written explicitly in terms of one variable. Next students will simply apply the rules they have already developed to find higher order derivatives. As students saw earlier, if given a position function, they can find the associated velocity function by determining the derivative of the position function. They can also take the second derivative of the position function and create a rate of change of velocity function that is more commonly referred to as the acceleration function which is where this unit ends.

Specific Expectations: A3.1, A3.2, A3.3, A3.4, A3.5

Unit 3: Applications of Derivatives and Related Rates

A variety of types of problems exist in this unit and are generally grouped into the following categories: Pythagorean Theorem Problems (these include ladder and intersection problems), Volume Problems (these usually involve a 3-D shape being filled or emptied), Trough Problems, Shadow problems and General Rate Problems. During this unit students will look at each of these types of problems individually.

Specific Expectations: B1.1. B1.2, B1.3, B1.4, B2.1. B2.2, B2.3, B2.4, B2.5, B2.6, B2.7

Unit 4: Curve Sketching

In Curve Sketching In previous math courses, functions were graphed by developing a table of values and smooth sketching between the values generated. This technique often hides key detail of the graph. In this unit, students will make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching; Solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.

Specific Expectations: B1.1. B1.2, B1.3, B1.4

Unit 5: Derivatives of Exponential and Trigonometric Functions

This unit begins with examples and exercises involving exponential and logarithmic functions using Euler's number (e). But as students have already seen, many other bases exist for exponential and logarithmic functions. Students will now look at how they can use their established rules to find the derivatives of such functions. The next topic should be familiar as the steps involved in sketching a curve that contains an exponential or logarithmic function are identical to those taken in the curve sketching unit studied earlier in the course. Because the derivatives of some functions cannot be determined using the rules established so far in the course, students will need to use a technique called logarithmic differentiation which is introduced

Specific Expectations: A2.1, A2.2, A2.3, A2.4, A2.5, A2.6, A2.7, A2.8, B2.1. B2.2, B2.3, B2.4, B2.5, B2.6, B2.7

Unit 6: Vectors: An Introduction

There are four main topics pursued in this initial unit of the course. These topics are: an introduction to vectors and scalars, vector properties, vector operations and plane figure properties. Students will tell the difference between a scalar and vector quantity, they will represent vectors as directed line segments and perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. Students will conclude the first half of the unit by proving some properties of plane figures, using vector methods and by modeling and solving problems involving force and velocity. Next students learn to represent vectors as directed line segments and to perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. The final topic involves students in proving some properties of plane figures using vector methods

Specific Expectations: C1.1, C1.2, C1.3, C1.4, C2.1, C2.2, C2.3, C2.4, C2.5, C2.6, C2.7, C2.8

Unit 7: Applications of Vectors

Cartesian vectors are represented in two-space and three-space as ordered pairs and triples, respectively. The addition, subtraction, and scalar multiplication of Cartesian vectors are all investigated in this unit. Applications involving work and torque are used to introduce and lend context to the dot and cross products of Cartesian vectors. The vector and scalar projections of Cartesian vectors are written in terms of the dot product. The properties of vector products are investigated and proven. These vector products will be revisited to predict characteristics of the solutions of systems of lines and planes in the intersections of lines and planes.

Specific Expectations: C1.1, C1.2, C1.3, C1.4, C2.1, C2.2, C2.3, C2.4, C2.5, C2.6, C2.7, C2.8

Unit 8: Equations of Lines and Planes

This unit begins with students determining the vector, parametric and symmetric equations of lines in R2 and R3 . Students will go on to determine the vector, parametric, symmetric and scalar equations of planes in 3-space. The intersections of lines in 3-space and the intersections of a line and a plane in 3-space are then taught. Students will learn to determine the intersections of two or three planes by setting up and solving a system of linear equations in three unknowns. Students will interpret a system of two linear equations in two unknowns geometrically, and relate the geometrical properties to the type of solution set the system of equations possesses. Solving problems involving the intersections of lines and planes, and presenting the solutions with clarity and justification forms the next challenge. As work with matrices continues students will define the terms related to matrices while adding, subtracting, and multiplying them. Students will solve systems of linear equations involving up to three unknowns, using row reduction of matrices, with and without the aid of technology and interpreting row reduction of matrices as the creation of new linear systems equivalent to the original constitute the final two new topics of this important unit.

Specific Expectations: C4.1, C4.2, C4.3, C4.4, C4.5, C4.6, C4.7

Unit 9: Relationships Between Points, Lines, and Plans

In this unit, we introduce perhaps the most important idea associated with vectors, the solution of systems of equations. In previous unit, the solution of systems of equations was introduced in situations dealing with two equations in two unknowns. Geometrically the solution of two

equations in two unknowns is the point of intersection between two lines on the xy-plane. In this unit, we are going to extend these ideas and consider systems of equations and interpret their meaning. Students will be working with systems up to three equations in three unknowns and will demonstrate techniques for solving these systems

Specific Expectations: C3.1, C3.2, C3.3, C4.1, C4.2, C4.3, C4.4, C4.5, C4.6, C4.7

Teaching and Learning Strategies

Effective instruction is key to student success and students learn best when they are engaged in a variety of ways of learning. Teachers at Insight Academy of Canada (IAC) provide numerous opportunities and use a variety of instructional, assessment, and evaluation strategies to help students develop skills of inquiry, problem solving, and communication as they investigate and learn fundamental concepts. The activities offered enable students not only to make connections among these concepts throughout the course but also to relate and apply them to relevant societal, environmental, and economic contexts. Opportunities to relate knowledge and skills to these wider contexts will motivate students to learn and to become lifelong learners.

The following seven mathematical process expectations describe a set of skills that support lifelong learning in mathematics and that students need to develop on an ongoing basis, as they work to achieve the expectations outlined within the course.

- Problem Solving: develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- Reasoning and Proving: develop and apply reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
- Reflecting: demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem;
- Selecting Tools and Computational Strategies: select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- Connecting: make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts;
- Representing: create a variety of representations of mathematical ideas, connect and compare them, and select and apply the appropriate representations to solve problems;
- Communicating: communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Each unit of the course contains a Unit Overview, a number of lessons, a Unit Review Assignment, a Unit Quiz, a Unit Test, and a Unit Exit Card. Lessons are delivered through the following format.

- Mind on. Students are introduced to the content through a variety of exploratory and instructional strategies including watching online videos.
- Actions. Students practice and apply their new learning through worked examples, investigation, and exploration. Students are actively engaged in their assessment process as they monitor their own learning to determine their next steps and set individual learning goals.
- Consolidation. Students are provided opportunities to demonstrate what they have learned through independent practice, reflection assignments, and discussion posts (Exit Slip).

- Extension Activities. Students study extra lesson resources, complete homework assignment, and response to teacher's follow-up questions to expand their learning and prepare for unit test.

A variety of teaching and learning strategies will be used in this course.

Direct Instruction Strategies • Online lecture • Video/Animation • Examples of full solutions • Scaffolding • Administering probes and/or prompts • Providing descriptive feedback • Providing pictorial or diagram presentation • Allowing independent practice and individually paced instruction • Teacher modelling • Providing individual instruction • Class activity, practice • Class and 1:1 Discussion/Chat • Student-Teacher Conferences • Q&A • Review, Seminar • Virtual Office Hours	**Independent Learning Strategies** • Homework Q&A • Handout, Work and Task Sheet • Class Investigations • Independent Study (teacher direction) • Self-Assessment • Self-regulation • Self-reflection • Exit Card • Survey • Learning Log • ePortfolio • Computer-Assisted Instruction • Interactive Online Activity • Student Exploration Tasks • Simulation • Graphing Software
Cooperative Strategies • Discussion Boards • Group/Peer Discussion (Think-Pair-Share) • Emails • Peer Conferencing • Peer Assessment • Peer Feedback	**Thinking-Skills Strategies** • Opinion Sharing/Commentary Offering • Oral Explanation • Presentation • Problem-Based Learning • Problem Solving • Reasoning and Proving • Reflective Thinking • Research Process • Case Study

Strategies for Assessment & Evaluation of Student Performance

Insight Academy of Canada's (IAC) Assessment and Evaluation policy is aligned with the Ministry of Education's Growing Success policy document which outlines the assessment, evaluation, and reporting policies and practices in Ontario schools.

Basic Considerations

The primary purpose of assessment and evaluation is to improve student learning. Assessment is the process of gathering information from a variety of sources that accurately reflects how well a student is achieving the curriculum expectations in a course. Evaluation refers to the process of

judging the quality of student learning on the basis of established performance standards, and assigning a value to represent that quality.

In order to ensure that assessment and evaluation are valid and reliable, and that they lead to the improvement of student learning, IAC teachers will use assessment and evaluation strategies that:

- are fair, transparent, and equitable for all students;
- support all students, including those with special education needs, those who are learning the language of instruction (English or French), and those who are First Nation, Métis, or Inuit;
- are carefully planned to relate to the curriculum expectations and learning goals and, as much as possible, to the interests, learning styles and preferences, needs, and experiences of all students;
- are communicated clearly to students and parents at the beginning of the school year or course and at other appropriate points throughout the school year or course;
- are ongoing, varied in nature, and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning;
- provide ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement;
- develop students' self-assessment skills to enable them to assess their own learning, set specific goals, and plan next steps for their learning

Evaluation and Reporting of Student Achievement

Insight Academy of Canada's (IAC) will use the Provincial Report Card, Grades 9–12, for formal written reports to students and parents two times a term. The report card provides a record of the student's achievement of the curriculum expectations in the course, at particular points in the school year or term, in the form of a percentage grade. The percentage grade represents the quality of the student's overall achievement of the expectations for the course and reflects the corresponding level of achievement as described in the achievement chart for the discipline.

A final grade is recorded for the course, and a credit is granted and recorded for the course in which the student's grade is 50% or higher. The final grade for the course will be determined as followings:

- Seventy per cent of the grade will be based on evaluations conducted throughout the course. This portion of the grade will reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement.
- Thirty per cent of the grade will be based on a final evaluation in the form of an examination and administered at the end of the course.

Assessment and Evaluation Categories and Weights			
Achievement Categories	**Percent**	**Evaluation for Final Grade**	**Percent**
Knowledge/Understanding	25%	Term Work	70%
Inquiry/Thinking	25%		
Communication	25%	Final Evaluation	30%
Application	25%		

Reporting on Demonstrated Learning Skills & Work Habits

The report card provides a record of the learning skills demonstrated by the student in every course, in the following six categories: Responsibility, Organization, Independent Work, Collaboration, Initiative & Self-regulation. These learning skills and work habits are evaluated using a four-point scale (E-Excellent, G-Good, S-Satisfactory, N-Needs Improvement). The separate evaluation and reporting of the learning skills and work habits in these six areas reflect their critical role in students' achievement of the curriculum expectations. To the extent possible, the evaluation of learning skills and work habits, apart from any that may be included as part of a curriculum expectation in a course, should not be considered in the determination of percentage grades.

In order to ensure that assessment and evaluation are valid and reliable, and that they lead to the improvement of student learning, I.A.C teachers use a variety of strategies throughout the course.